Càlcul integral

Exercicis d'integració indefinida 2

solucions

Integrals immediates:

1) $\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}}dx =$ $3e^{\sqrt[3]{x}} + C$

2) $\int \frac{4^{arctan(x)}}{1+x^2}dx =$ $\frac{1}{\ln 4}4^{arctan(x)} + C$

3) $\int \frac{1}{x^2}tan\left(\frac{1}{x}\right)dx =$ $\ln\left|\cos\frac{1}{x}\right| + C$

4) $\int \frac{e^{4x}}{1+e^{4x}}dx =$ $\frac{1}{4}\ln\left|1+e^{4x}\right| + C$

5) $\int \frac{1}{sin(x)}dx =$ $\ln\left|tan\left(\frac{x}{2}\right)\right| + C$

6) $\int x\left(x^2+1\right)^{3/7}dx =$ $\frac{7}{20}\left(x^2+1\right)^{10/7} + C$

7) $\int \left(e^x+1\right)^3 e^x dx =$ $\frac{1}{4}\left(e^x+1\right)^4 + C$

8) $\int \frac{sin(2x)}{\sqrt[3]{5+sin^2(x)}}dx =$ $\frac{3}{5}\left(sin^2(x)+5\right)^{2/3} + C$

9) $\int \frac{1}{x^3}\cos\left(\frac{1}{x^2}\right)dx =$ $-\frac{1}{2}sin\left(\frac{1}{x^2}\right) + C$

10) $\int x^3\cos\left(2x^4+1\right)dx =$ $\frac{1}{8}sin\left(2x^4+1\right) + C$

11) $\int 3^{2x}sin\left(1+3^{2x}\right)dx =$ $-\frac{1}{2\ln(3)}\cos\left(1+3^{2x}\right) + C$

12) $\int \frac{1}{\sqrt{x}\cos^2\left(\sqrt{x}\right)}dx =$ $2\ tan\left(\sqrt{x}\right) + C$

13) $\int \frac{1}{e^x\cos^2\left(e^{-x}\right)}dx =$ $-tan\left(e^{-x}\right) + C$

14) $\int \frac{1}{x\ sin^2\left(1+\ln x\right)}dx =$ $-\cot an\left(1+\ln(x)\right) + C$

15) $\int \frac{x^2}{\sqrt{1-x^6}}dx =$ $\frac{1}{3}arcsin\left(x^3\right) + C$

16) $\int \frac{1}{\sqrt{4-x^2}}dx =$ $arcsin\left(\frac{x}{2}\right) + C$

càlcul integral

17) $\int \frac{1}{\sqrt{-4x-x^2}}dx =$ $\quad arcsin\left(\frac{x+2}{2}\right)+C$

18) $\int \frac{1}{\sqrt{20+8x-x^2}}dx =$ $\quad arcsin\left(\frac{x-4}{6}\right)+C$

19) $\int \frac{1}{\sqrt{5-8x-3x^2}}dx =$ $\quad \frac{\sqrt{3}}{3}arcsin\left(\frac{3x+4}{\sqrt{31}}\right)+C$

20) $\int \frac{1}{9+x^2}dx =$ $\quad \frac{1}{3}arctan\left(\frac{x}{3}\right)+C$

21) $\int \frac{1}{4x^2+9}dx =$ $\quad \frac{1}{6}arctan\left(\frac{2x}{3}\right)+C$

22) $\int \frac{1}{x^2+10x+30}dx =$ $\quad \frac{1}{\sqrt{5}}arctan\left(\frac{2x}{3}\right)+C$

23) $\int \frac{1}{2x^2+5x+3}dx =$ $\quad \frac{2\sqrt{79}}{79}arctan\left(\frac{4x+5}{\sqrt{79}}\right)+C$

24) $\int \frac{e^x}{e^{2x}+6e^x+10}dx =$ $\quad arctan\left(e^x+3\right)+C$

25) $\int x^4 ch(x^5)dx =$ $\quad \frac{1}{5}sh(x^5)+C$

26) $\int x^{-1/2} sh(\sqrt{x})dx =$ $\quad 2\ ch(\sqrt{x})+C$

27) $\int \frac{ch(\ln(x))}{x}dx =$ $\quad sh(\ln(x))+C$

28) $\int \frac{x}{ch^2(4+3x^2)}dx =$ $\quad \frac{1}{6}Th(4+3x^2)+C$

29) $\int \frac{1}{\sqrt{4x^2+49}}dx =$ $\quad \frac{1}{2}\arg Sh\left(\frac{2x}{7}\right)+C$

30) $\int \frac{1}{\sqrt{x^2+6x+21}}dx =$ $\quad \arg Sh\left(\frac{x+3}{\sqrt{12}}\right)+C$

31) $\int \frac{1}{\sqrt{x^2+6x+1}}dx =$ $\quad \arg Ch\left(\frac{x+3}{\sqrt{8}}\right)+C$

32) $\int \frac{1}{\sqrt{3x^2-5x+16}}dx =$ $\quad \arg Sh\left(\frac{6x-5}{\sqrt{167}}\right)+C$

33) $\int \frac{1}{4x^2-3x-10}dx =$ $\quad -\frac{2}{13}\arg Th\left(\frac{8x-3}{13}\right)+C$

34) $\int \frac{1}{x^2-8x-3}dx =$ $\quad -\frac{1}{\sqrt{19}}\arg Th\left(\frac{x-4}{13}\right)+C$

Integrals per descomposició:

35) $\int \mathbf{sin}^2(x)\,dx =$ x/2 - 1/4 **sin**(2x) +C

36) $\int \mathbf{cos}^2(x)\,dx =$ x/2 + 1/4 **sin**(2x) +C

37) $\int \mathbf{tan}^2(x)\,dx =$ **tan**(x) -x +C

38) $\int \frac{1}{\mathbf{sin}^2(x)\ \mathbf{cos}^2(x)}\,dx =$ **tan**(x) - **cotan**(x) +C

39) $\int \frac{x}{(x+1)^2+1}\,dx =$ 1/2 **ln**| $(x+1)^2$ +1 | - **arcsin**(x+1) +C

40) $\int \sqrt{\frac{1+x}{1-x}}\,dx =$ **arcsin**(x) - $(1-x^2)^{1/2}$ +C

41) $\int \left(1+\sqrt{x}\right)\left(1+x-\sqrt{x}\right)dx =$ $x+\frac{2}{5}x^2\sqrt{x}$ +C

42) $\int \left(x^2+1\right)\left(x^2-2\right)x^{-2/3}dx =$ $\frac{3}{13}x^4\sqrt[3]{x}-\frac{3}{7}x^2\sqrt[3]{x}$ +C

43) $\int \frac{\left(\sqrt{a}-\sqrt{x}\right)^4}{\sqrt{ax}}dx =$ $2a^{3/2}\sqrt{x}-4ax+4a^{1/2}x^{3/2}-2x^2+\frac{2}{5}a^{-1/2}x^{5/2}$ +C

44) $\int \frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}\,dx =$ $\mathbf{arcsen}\left(\frac{x}{2}\right)-\mathbf{arg}\,Sh\left(\frac{x}{2}\right)$ +C

45) $\int \frac{x+2}{x+1}dx =$ $x+\mathbf{ln}|x+1|$ +C

46) $\int \frac{x+3}{\sqrt{1-x^2}}dx =$ $-\sqrt{1-x^2}+3\,\mathbf{arcsin}(x)$ +C

47) $\int \frac{2x-7}{x^2+9}\,dx =$ **ln**| x^2+9 | - 7/3 **arctan**(x/3) +C

48) $\int \frac{x+1}{x^2-4x+8}\,dx =$ 1/2 **ln**| x^2-4x+8 | + 3/2 **arctan**(1/2 (x-2)) +C

49) $\int \frac{2x+3}{9x^2-12x+18}\,dx =$ 1/9 **ln**| $9x^2$-12x+8 | + 13/28 **arctan**(1/2 (3x-2)) +C

50) $\int \frac{x+2}{\sqrt{4x-x^2}}\,dx =$ -$(4x-x^2)^{1/2}$ + 4 **arcsin**(1/2 (x-2)) +C

càlcul integral

51) $\int \frac{x^2 - 5x + 6}{x^2 + 4} dx =$ x- 5/2 **ln**| x^2+4 | + **arctan**(x/2) +C

52) $\int \frac{x - \sqrt{\mathbf{arctan}(2x)}}{1 + 4x^2} dx =$ 1/8 **ln**| $1+4x^2$ | - 1/3 (**arctan**(2x))$^{3/2}$ +C

53) $\int \mathbf{sin}^2(ax)\ dx =$ 1/2 x - 1/(4a) **sin**(2ax) +C

54) $\int \mathbf{cos}^2(ax)\ dx =$ 1/2 x + 1/(4a) **sin**(2ax) +C

55) $\int \mathbf{sin}^2(x)\ \mathbf{cos}^3(x)\ dx =$ 1/3 **sin**^{3}(x) - 1/5 **sin**^{5}(x) +C

56) $\int \mathbf{sin}^3(3x)\ \mathbf{cos}^5(3x)\ dx =$ -1/18 **cos**^{6}(3x) + 1/24 **cos**^{8}(3x) +C

57) $\int \mathbf{cos}(6x + 7)\ \mathbf{cos}(2x - 5)\ dx =$ 1/16 sin(8x+2) + 1/8 sin(4x+12) + C

58) $\int \sin(3x)\ \mathbf{cos}(5x)\ dx =$ -1/4 **cos**(8x) + 1/4 **cos**(2x) + C

59) $\int \mathbf{sin}(9x - 1)\ \mathbf{sin}(2x - 5)\ dx =$ 1/14 **sin**(7x-6) - 1/22 **sin**(11x+4) + C

60) $\int \mathbf{cos}(7x)\ \mathbf{sin}(3x)\ \mathbf{cos}(5x)\ dx =$ -1/60 **cos**(15x) + 1/36 **cos**(9x) - 1/20 **cos**(5x)+ C

61) $\int \frac{\mathbf{sin}(2x + 5)}{\mathbf{cos}(2x)} dx =$ -1/2 **cos**(5) **ln**| **cos**(2x) | + x **sin**(5) + C

62) $\int \frac{x^5 - x^4 + 2x^3 - 7x^2 + x - 6}{(x - 1)^7} dx =$ 5/3 $(x-1)^{-6}$ + 6/5 $(x-1)^{-5}$ - 3/4 $(x-1)^{-4}$ - 8/3 $(x-1)^{-3}$ -2$(x-1)^{-2}$ - $(x-1)^{-1}$

63) $\int$ **sin**(3x) **cos**(5x) dx = 1/4 **cos**(2x) - 1/16 **cos**(8x) +C

64) $\int$ **cos**(4x) **cos**(2x) dx = 1/4 **sin**(2x) + 1/12 **sin**(6x) +C

65) $\int$ **sin**(3x) **sin**(2x) dx = 1/2 **sin**(x) - 1/10 **sin**(5x) +C

66) $\int$ **tan**^{5}(x) dx = 1/4 **tan**^{4}(x) - 1/2 **tan**^{2}(x) + **ln**| **sec**(x) | +C

67) $\int$ **tan**^{3}(3x) **sec**^{4}(3x) dx = 1/12 **tan**^{4}(3x) + 1/18 **tan**^{6}(3x) +C

68) $\int$ **cotan**^{3}(x) **cosec**^{5}(x) dx = -1/7 **cosec**^{7}(x) + 1/5 **cosec**^{5}(x) +C

Integrals per parts:

69) $\int \ln(x)\,dx =$ $x\ln|x| - x + C$

70) $\int \arctan(x)\,dx =$ $x\arctan(x) - \ln[(1-x^2)^{1/2}] + C$

71) $\int x^2\cos(2x)\,dx =$ $x^2/2\sin(2x) + x/2\cos(2x) - 1/4\sin(2x) + C$

72) $\int x\sqrt{1+x}\,dx =$ $\frac{2}{3}x(1+x)^{3/2} - \frac{4}{15}(1+x)^{5/2} + C$

73) $\int x^2\ln(x)dx =$ $\frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3 + C$

74) $\int x^2 sin(x)dx =$ $-x^2\cos(x) + 2xsin(x) + 2\cos(x) + C$

75) $\int x^2\sqrt{1-x}\,dx =$ $-\frac{2}{3}x^2(1-x)^{3/2} - \frac{8}{15}x(1-x)^{5/2} - \frac{16}{105}(1-x)^{7/2} + C$

76) $\int x\,arcsin(x^2)dx =$ $\frac{1}{2}x^2 arcsin(x^2) + \frac{1}{2}\sqrt{1-x^4} + C$

77) $\int sin(\ln(x))dx =$ $\frac{1}{2}[xsin(\ln(x)) - x\cos(\ln(x))] + C$

78) $\int x^3 e^{2x}\,dx =$ $\frac{1}{2}e^{2x}\left[x^3 - \frac{3}{2}x^2 + \frac{3}{2}x - \frac{3}{4}\right] + C$

79) $\int \frac{\ln(x)}{(x+3)^2}\,dx =$ $-\ln(x) - \frac{2}{3}\arg Th\left(\frac{2}{3}x + 1\right) + C$

80) $\int \arcsin\left(\frac{1}{x}\right)dx =$ $xarcsin\left(\frac{1}{x}\right) + \arg Ch(x) + C$

81) $\int xe^{2x}\cos(3x)\,dx =$ $\frac{1}{7}e^{2x}\left[x\left(sin(3x) + \frac{2}{3}\cos(3x)\right) - \frac{18}{3}sin(3x) - \frac{1}{13}\cos(3x)\right] + C$

82) $\int \arcsin(x)\,dx =$ $\left(xarcsin(x) + \sqrt{1-x^2}\right) + C$

83) $\int x\arctan(x)\,dx =$ $\frac{1}{2}\left(x^2 arctan(x) - x + arctan(x)\right) + C$

84) $\int \arcsin\left(\sqrt{\frac{x}{x+1}}\right)dx =$ $(x+1)\arctan[(x/(x+1))^{1/2}] - x^{1/2} + C$

85) $\int e^x\cos(x)\,dx =$ $\frac{1}{2}e^x(\cos(x) + sin(x)) + C$

Càlcul integral

86) $\int e^x \sin(x)\,dx =$ $\frac{1}{2}e^x\left(sin(x)-\cos(x)\right)+C$

87) $\int \frac{xe^x}{(1+x)^2}\,dx =$ $-\frac{xe^x}{1+x}+e^x+C$

88) $\int \frac{\arcsin(\sqrt{x})}{\sqrt{x}}\,dx =$ $\frac{\sqrt{x}}{2}\,\mathbf{arcsin}(\sqrt{x})+\frac{1}{2}\sqrt{1-x}+C$

89) $\int \sin^2(x)\,dx =$ 1/2 x - 1/2 **sin**(x) **cos**(x) +C

90) $\int e^{ax}\cos(bx)\,dx =$ $a/b^2\ e^{ax}$ **cos**(bx) + 1/b e^{ax} **sin**(bx) +C

91) $\int x\arctan\left(\frac{1}{1+x}\right)dx =$ 1/2 [x^2 **arctan**(1/(1+x)) + x - **ln**| x^2 +2x +2 | +C

92) $\int \left(\frac{\mathbf{ln}(x+1)}{\sqrt{x+1}}\right)dx =$ **ln**| x+1 | 2 $(x+1)^{1/2}$ - 4 $(x+1)^{1/2}$ +C

Integrals per substitució:

93) $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx =$ $\left[x=\frac{1}{t}\right]$ $-(1-x^2)^{1/2}/x$ +C

94) $\int \frac{x}{\sqrt{2+x}}\,dx =$ $[2+x=t^2]$ 2/3 $(2+x)^{3/2}$ -4 $(2+x)^{1/2}$ +C

95) $\int \frac{x^3}{\left(4+x^2\right)^3}\,dx =$ [x=2**Sh**(t)] 1/16 $x/(4+x^2)^{1/2}$+C

96) $\int \sqrt[4]{(3-2x)^3}\,dx =$ [3-2x=t] -2/7 $(3-2x)^{7/4}$+C

97) $\int \frac{1}{x^2\sqrt{4+x^2}}\,dx =$ $\left[x=\frac{1}{t}\right]$ $-\frac{1}{4}\frac{\sqrt{4+x^2}}{x}$ +C

98) $\int \frac{x^2}{\sqrt{x^2-4}}\,dx =$ [x=**Ch**(t)] x/2 $(x^2-4)^{1/2}$ + 2 **argCh**(x/2) +C

99) $\int \mathbf{sin}\left(\sqrt{x}\right)dx =$ $[x=t^2]$ p -2 $x^{1/2}$ **cos**($x^{1/2}$) + 2 **sin**($x^{1/2}$) +C

100) $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx =$ $\left[x=\frac{1}{t}\right]$ $-\frac{1}{4}\frac{\sqrt{4-x^2}}{x}$ +C

101) $\int \frac{e^x\left(e^x-2\right)}{e^x+1}dx =$ [$e^x=t$] $e^x+1-3\ \ln(e^x+1)+C$

102) $\int x^5\sqrt{1-x^3}\,dx =$ [$1-x^3=t$] $-2/9\ (1-x^3)^{3/2} + 2/15\ (1-x^3)^{5/2} +C$

103) $\int \frac{1}{e^x+1}dx =$ [$e^x=t$] $-\ln|\ 1+e^{-x}\ | +C$

104) $\int \frac{\ln(2x)}{\ln(4x)}dx =$ [$\ln(2x)=t$] $\ln|\ 4x\ | - \ln(2)\ \ln|4x| + C$

105) $\int \frac{1}{\sqrt{e^x-1}}dx =$ [$e^x-1=t^2$] $2\ \arctan(\ (e^x-1)^{1/2}\ ) +C$

106) $\int \frac{e^{2x}}{\sqrt{e^x+1}}dx =$ [$e^x+1=t^2$] $2/3\ (\ e^x+1)^{3/2} - 2\ (\ e^x+1)^{1/2} +C$

107) $\int \frac{\sin^3 x}{\sqrt{\cos x}}dx =$ [$\cos(x)=t$] $2\ \cos^{1/2}(x) - 2/5\ \cos^{5/2}(x) +C$

108) $\int \frac{x\bullet e^{\arcsin(x)}}{\sqrt{1-x^2}}dx =$ [$\arcsin(x)=t$] $e^{\arcsin(x)}\ (\ x/2 - 1/2\ (1-x^2)^{1/2}\ ) +C$

109) $\int \frac{\sqrt[3]{x}+\sqrt{x}}{1+\sqrt[3]{x}}dx =$ [$x^{1/6}=t$] $1/2\ x^{1/3} + x^{1/6} - 1/2\ \ln|1+x^{1/3}| - \arctan(x^{1/6}) + C$

110) $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx =$ [$x^{1/2}=t$] $4/3\ (\ 1+ x^{1/2}\ )^{3/2} +C$

Integrals de funcions racionals:

111) $\int \frac{\sqrt{x}}{x+2}\,dx =$ $-\frac{1}{6}\ln|x| + \frac{3}{10}\ln|x-2| - \frac{2}{15}\ln|x+3| +C$

112) $\int \frac{x^2+3x-4}{x^2-2x-8}\,dx =$ $x + \ln|x+2| + 4\ln|x-4| +C$

113) $\int \frac{x^2-3x-1}{x^3+x^2-2x}\,dx =$ $\frac{1}{2}\ln|x| + \frac{3}{2}\ln|x+2| - \ln|x-1| +C$

114) $\int \frac{5x^3+2}{x^3-5x^2+4x}\,dx =$ $5x + \frac{1}{2}\ln|x| - \frac{7}{2}\ln|x-1| + \frac{161}{6}\ln|x-4| +C$

115) $\int \frac{x^2+4}{6x^3-5x^2-6x}\,dx =$ $-\frac{2}{3}\ln|x| + \frac{25}{78}\ln|2x-3| + \frac{20}{39}\ln|3x+2| +C$

càlcul integral

116) $\int \frac{4x^2-5x+2}{(4x^2-1)(5x-2)}\,dx = \frac{1}{4}\ln|2x-1| - \frac{11}{36}\ln|2x+1| - \frac{16}{45}\ln|5x-2| + C$

117) $\int \frac{3x+5}{x^3-x^2-x+1}\,dx = -\frac{4}{x-1} + \frac{1}{2}\ln|x+1| - \frac{1}{2}\ln|x-1| + C$

118) $\int \frac{x}{(x-2)^2}\,dx = \ln|x-2| - \frac{2}{x-2} + C$

119) $\int \frac{x^4}{(1-x)^3}\,dx = -\frac{1}{2}x^2 - 3x - 6\ln|1-x| + \frac{1}{2(1-x)^2} - \frac{4}{1-x} + C$

120) $\int \frac{x^4-x^3-x+1}{x^3-x^2}\,dx = \frac{x^2}{2} + 2\ln|x| - \frac{1}{x} - 2\ln|x-1| + C$

121) $\int \frac{1}{x(x+1)^2}\,dx = \frac{1}{x+1} + \ln|x| - \ln|x+1| + C$

122) $\int \frac{x^4-6x^3+12x^2+6}{x^3-6x^2+12x-8}\,dx = \frac{1}{2}x^2 - \frac{11}{(x-2)^2} - \frac{8}{x-2} + C$

123) $\int \frac{5x^2+6x+9}{(x-3)^2(x+1)^2}\,dx = -\frac{9}{2(x-3)} - \frac{1}{2(x+1)} + C$

124) $\int \frac{x^4-2x^3+3x^2-x+3}{x^3-2x^2+3x}\,dx = \frac{1}{2}x^2 + \ln|x| - \frac{1}{2}\ln|x^2-2x+3| + C$

125) $\int \frac{x^3+x^2+x+3}{(x^2+1)(x^2+3)}\,dx = arctan(x) + \frac{1}{2}\ln|x^2+3| + C$

126) $\int \frac{x^4+8x^3-x^2+2x+1}{(x^2+x)(x^3+1)}\,dx = \ln|x| - \frac{3}{x+1} - 2\ln|x+1| + \ln|x^2-x+1| + \frac{2\sqrt{3}}{3}arctan\left(\frac{2x-1}{\sqrt{3}}\right) + C$

127) $\int \frac{x^3+x^2-5x+15}{(x^2+5)(x^2+2x+3)}\,dx = \frac{1}{2}\ln|x^2+2x+3| + \frac{5\sqrt{2}}{2}arctan\left(\frac{x+1}{\sqrt{2}}\right) - \sqrt{5}arctan\left(\frac{x}{\sqrt{5}}\right) + C$

128) $\int \frac{1}{(x^2-4x+3)(x^2+4x+5)}\,dx = \frac{1}{52}\ln|x-3| - \frac{1}{x20}\ln|x-1| + \frac{1}{65}\ln|x^2+4x+5| + \frac{7}{130}arctan(x+2) + C$

129) $\int \frac{1}{x^4+1}\,dx = \frac{1}{4\sqrt{2}}\ln\left|\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right| + \frac{1}{2\sqrt{2}}arctan\left(x\sqrt{2}-1\right) + \frac{1}{2\sqrt{2}}arctan\left(x\sqrt{2}+1\right) + C$

130) $\int \frac{x^4}{x^4-1}\,dx = x + \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}arctan(x) + C$

131) $\int \frac{1}{x^3+1}\,dx = \frac{1}{6}\ln\left|\frac{(x+1)^2}{x^2-x+1}\right| - \frac{1}{\sqrt{3}}arctan\left(\frac{2x-1}{\sqrt{3}}\right) + C$

132) $\int \frac{1}{(x^2+1)^2}\,dx = \frac{1}{2}arctan(x) + \frac{x}{2(x^2+1)} + C$

133) $\int \frac{3x+5}{\left(x^2+2x+2\right)^2}\,dx =$ $\frac{2x-1}{2\left(x^2+2x+2\right)^2}+arctan(x)+C$

Integrals de funcions racionals: Mètode d'Hermite:

134) $\int \frac{1}{(x+1)^2\left(x^2+1\right)^2}\,dx = \frac{-x^2+x}{4(x+1)(x^2+1)}+\frac{1}{2}\ln|x+1|-\frac{1}{4}\ln|x^2+1|+\frac{1}{4}\arctan(x)+C$

135) $\int \frac{1}{(x+1)\left(x^2+x+1\right)^2}\,dx = \frac{x+2}{3\left(x^2+x+1\right)}+\ln|x+1|+\frac{5}{3\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\ln|x^2+x+1|+C$

136) $\int \frac{x^3+1}{\left(x^2-4x+5\right)^2}\,dx = \frac{3x-17}{2(x^2-4x+5)}+\frac{1}{2}\ln|x^2-4x+5|+\frac{5}{2}\arctan(x-2)+C$

137) $\int \frac{1}{\left(x^2+1\right)^4}\,dx =$ $\frac{15x^5+40x^3+33x}{48\left(1+x^2\right)^3}+\frac{5}{16}\arctan(x)+C$

138) $\int \frac{x^2}{\left(x^2+1\right)^2}\,dx =$ $-\frac{1}{2}\frac{x}{\left(x^2+1\right)}+\frac{1}{2}\arctan(x)+C$

139) $\int \frac{1}{\left(x^2-1\right)^4}\,dx =$ $-\frac{x}{4\left(x^4-1\right)}-\frac{3}{8}\arctan(x)-\frac{3}{16}\ln\left|\frac{x-1}{x+1}\right|+C$

Integrals de funcions irracionals algebraiques:

140) $\int \frac{\sqrt{x}}{x+2}\,dx = [x^{1/2}=t]$ $2x^{1/2}-2^{3/2}\arctan(x/2)^{1/2}+C$

141) $\int \frac{\sqrt{x}-1}{\sqrt[3]{x}+1}dx = [x=t^6]$ $6/7\,x^{7/6}-6/5x^{5/6}-3/2x^{2/3}+2x^{1/2}-3x^{1/3}-6x^{1/6}-3\ln|1+x^{1/3}|+6\arctan(x^{1/6})+C$

142) $\int \frac{1}{(2-x)\sqrt{1-x}}dx = [1-x=t^2]$ $-2\arctan(1-x)^{1/2}+C$

143) $\int \frac{x+3}{x^2\sqrt{2x+3}}dx = [2x+3=t^2]$ $-1/x\,(2x+3)^{1/2}+C$

144) $\int x\sqrt{\frac{x-1}{x+1}}\,dx = [(x-1)/(x+1)=t^2]$ $\frac{\sqrt{x^2-1}}{2}(x-2)+\frac{1}{2}\ln\left|x+\sqrt{x^2-1}\right|+C$

145) $\int \sqrt[3]{\frac{x+1}{x-1}}dx = [(x+1)/(x-1)=t^3]$ $\frac{1}{3}\ln\left|\frac{t^2+t+1}{t-1}\right|+\frac{2}{\sqrt{3}}\arctan\left(\frac{2t+1}{\sqrt{3}}\right)+\frac{2t}{t^3-1}+C$

146) $\int \frac{1}{x\sqrt{x^2-2x-3}}dx =$ $\frac{2}{\sqrt{3}}\arctan\left(\frac{\sqrt{x^2-2x-3}-x}{\sqrt{3}}\right)+C$

147) $\int \frac{x}{\sqrt{x^2+4x}}dx = [(x^2+4x)^{1/2}/x=t]$ $\frac{4t}{t^2-1}+2\ln\left|\frac{t-1}{t+1}\right|+C$

148) $\int \frac{1}{(x-1)\sqrt{x^2-3x+2}}\,dx =$ $\qquad \frac{-2}{1-x+\sqrt{x^2-3x+2}} + C$

149) $\int \frac{1}{\left(\sqrt{x^2-5x+6}\right)^3}\,dx =$ $\qquad \frac{4x+2}{\sqrt{x^2-5x+6}} + C$

Integrals de funcions del tipus: $\int \frac{A_n(x)}{\sqrt{ax^2+bx+c}}\,dx$

150) $\int \frac{-4x+10}{\sqrt{-x^2+4x-3}}\,dx =$ $\qquad 4(-x^2+4x-3)^{1/2} + 2\mathbf{arcsin}(x-2) + C$

151) $\int \frac{x^4+4x^2}{\sqrt{x^2+4}}\,dx =$ $\qquad \left(\frac{1}{4}x^3+\frac{1}{2}x\right)\sqrt{x^2+4} - 2\int \frac{1}{\sqrt{x^2+4}}\,dx$

152) $\int \frac{x^3}{\sqrt{-x^2+2x+1}}\,dx =$ $\qquad -\frac{19+15x+2x^2}{6}\sqrt{1+2x-x^2} - 4\,\mathbf{arcsin}\left(\frac{1-x}{\sqrt{2}}\right) + C$

153) $\int \frac{x^3-6x^2+11x-6}{\sqrt{x^2+4x+3}}\,dx = \left(\frac{x^2}{3}-\frac{14x}{3}+37\right)\sqrt{x^2+4x+3} - 66\,\mathbf{ln}\left|x+2+\sqrt{x^2+4x+3}\right| + C$

Nota $\int \frac{1}{(x-d)^n\sqrt{ax^2+bx+c}}\,dx$ amb el canvi x-d=1/t es converteix en un altra del tipus d'abans.

Integrals de funcions del tipus: $\int R\left(x,\sqrt{ax^2+bx+c}\right)dx$,

R funció racional

154) $\int \frac{1}{1-x^2+2\sqrt{1-x^2}}\,dx = [(1-x^2)^{1/2} = 1+tx\,]$ $\qquad -\frac{2}{\sqrt{3}}\arctan\left(\frac{\sqrt{1-x^2}-1}{x\sqrt{3}}\right) + C$

155) $\int \frac{1}{x\sqrt{x^2-x-4}}\,dx = [(x^2-x-4)^{1/2} = x+t\,]$ $\qquad \arctan\left(\frac{\sqrt{x^2-x-4}-x}{2}\right) + C$

156) $\int \frac{5}{\sqrt{3+2x-x^2}}\,dx = [(3+2x-x^2)^{1/2} = t(x+1)\,]$ $\qquad 2\arctan\left(\frac{\sqrt{3+2x-x^2}}{x+1}\right) + C$

Integrals binòmiques

157) $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx =$ (m+1)/n=2 $\qquad \frac{12}{7}\sqrt[3]{\left(1+\sqrt[4]{x}\right)^7} - 3\sqrt[3]{\left(1+\sqrt[4]{x}\right)^4} + C$

158) $\int x^3\left(1+2x^2\right)^{-3/2}\,dx =$ $\qquad \frac{1+x^2}{2\sqrt{1+2x^2}} + C$

159) $\int \frac{1}{x^4\sqrt{1+x^2}}\,dx =$ $\qquad \frac{\left(2x^2-1\right)\sqrt{1+x^2}}{3x^3} + C$

160) $\int \frac{1}{x^3\sqrt{1+x^5}}\,dx = \frac{1}{10}\ln\left(\frac{\left(\sqrt[3]{1+x^5}-1\right)^2}{\sqrt[3]{\left(1+x^5\right)^2}+\sqrt[3]{1+x^5}+1}\right)+\frac{\sqrt{3}}{5}\arctan\left(\frac{2\sqrt[3]{1+x^5}+1}{\sqrt{3}}\right)+C$

161) $\int x^{-2}\left(2+x^3\right)^{-5/3}dx = -\frac{4+3x^3}{8x\sqrt[3]{\left(2+x^3\right)^2}}+C$

162) $\int \frac{1}{\sqrt{x^3}\sqrt[3]{1+\sqrt[4]{x^3}}}\,dx = -2\sqrt[3]{\left(x^{-3/4}+1\right)^2}+C$

163) $\int \frac{1}{\sqrt[3]{x^2}\left(1+\sqrt[3]{x^2}\right)}\,dx = 3\arctan\left(\sqrt[3]{x}\right)+C$

164) $\int \frac{x^3}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}\sqrt{1-x^2}\left(x^2+2\right)+C$

165) $\int \frac{1+\arctan(x)}{\left(1-\arctan(x)\right)\left(1+x^2\right)}\,dx = -\frac{\sqrt{1+x^2}}{x}-\frac{x}{\sqrt{1+x^2}}+C$

Integrals de funcions transcendents:

166) $\int \frac{1+\arctan(x)}{\left(1-\arctan(x)\right)\left(1+x^2\right)}\,dx =$ [arctan(x)=t] $-\arctan(x)-2\ln|1-\arctan(x)|+C$

167) $\int \frac{\cos^3(x)}{1-\sin(x)}\,dx =$ [sin(x)=t] $\sin(x)+0.5\sin^2(x)+C$

168) $\int \sin^7(x)\,dx =$ [cos(x)=t] $-\cos(x)+\cos^3(x)-3/5\cos^5(x)+1/7\cos^7(x)+C$

169) $\int \frac{1}{\sin(x)\cos^2(x)}\,dx =$ [cos(x)=t] $0.5\ln|\cos(x)-1|-0.5\ln|\cos(x)+1|+\cos^{-1}(x)+C$

170) $\int \frac{dx}{\sin(x)+\cos(x)} =$ [tan(x/2)=t] $\sqrt{2}\,\mathrm{arg\,Th}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+C$

171) $\int \frac{dx}{8-4\sin(x)+7\cos(x)} =$ [tan(x/2)=t] $\ln\left|\frac{\tan\left(x/2\right)-5}{\tan\left(x/2\right)-3}\right|+C$

172) $\int \frac{\sin^2(x)}{\cos^6(x)}\,dx =$ [tan(x)=t] $\frac{1}{3}\tan^3(x)+\frac{1}{5}\tan^5(x)+C$

173) $\int \frac{1+\cos(x)}{1-\cos(x)}\,dx =$ [tan(x/2)=t] $-2\cot an(x/2)-x+C$

càlcul integral

174) $\int \frac{dx}{\cos(x)+2\sin(x)+3}dx =$	[tan(x/2)=t]	arctan(tan(x/2)+1)+C
175) $\int \frac{\cos(x)}{\sin(x)}dx =$	[sin(x)=t]	ln| sin(x) | +C
176) $\int \tan^3(x)\ dx =$	[tan(x)=t]	$0.5\tan^2(x)-0.5\ln\lvert\tan^2(x)+1\rvert+C$
177) $\int \frac{dx}{\tan(2x)+\sin(2x)} =$	[tan(x)=t]	$0.25\ln\lvert\tan(x)\rvert -0.125\tan^2(x)+C$
178) $\int \frac{\tan(x)}{1+\cos(x)}dx =$	[tan(x/2)=t]	$-\ln\lvert\cos(x)\rvert + \ln\lvert\cos(x)+1\rvert +C$
179) $\int \frac{\sec(x)}{\sec(x)+\tan(x)}dx =$	[tan(x/2)=t]	$\frac{-2}{\tan\left(x/2\right)+1}+C$
180) $\int \frac{e^{2x}}{\sqrt[4]{e^x+1}}dx =$	$[e^x+1=t^4]$	$\frac{4}{7}\sqrt[4]{\left(e^x+1\right)^7}-\frac{4}{3}\sqrt[4]{\left(e^x+1\right)^3}+C$
181) $\int \frac{dx}{e^{2x}-e^x} =$	$[e^x=t]$	$\frac{1}{2e^x}-\frac{x}{4}+\frac{1}{4}\ln\lvert e^x-2\rvert+C$
182) $\int \frac{e^x+1}{e^x-1}dx =$	$[e^x=t]$	$-\ln(e^x)+2\ln(e^x+1)+C$